Author's Reply to Comment by Kenneth Wang

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THE solution by Wang and Ting[1] was obtained by assuming V, contained in the term $[(gR_0/V^2) - 1]$, equal to the initial entry speed V_f, which is a constant. After initial portion of entry [say, $V/(gR_0)^{1/2}$ less than 1], V is quite different from the initial entry speed V_f; consequently, the solution no longer applies. The numerical values (0.986 and 10°32′) calculated by Wang[2] agree well with both the second-order solution[3] and the exact numerical solution, Figs. 8c and 8d, but they are for "from entry to skip," which is the initial portion of the overall re-entry trajectory dealt as a whole by the second-order theory.[3] When the second-order solution applies only to the initial portion of the overall re-entry trajectory [say, for $V/(gR_0)^{1/2} = 2^{1/2}$ to 1 or approximately 1], the second-order solution reduces to Wang's solution.[4]

References

[1] Wang, K. and Ting, L., "An approximate analytical solution of re-entry trajectory with aerodynamic forces," ARS J. **30,** 565–566 (1960).

[2] Wang, K., "Comment on 'A second-order theory of entry mechanics into a planetary atmosphere'," AIAA J. **1,** 977 (1963).

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[4] Loh, W. H. T., "Supercircular gliding entry," ARS J. **32,** 1398 (1962).

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Comment on "Stability of Pressure Waves in a Combustion Field"

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IN his recent technical note, Rosen[1] made an illuminating and welcome contribution to the field of combustion instability. However, several points are worthy of clarification. It is assumed in this note that the physical and chemical combustion processes are represented by a one-step rate controlling reaction that is a function of chamber conditions at the instant of reaction. It should be noted, however, that finite time delays occur and are important in actual combustion chambers. Although it is true that simple chemical reaction rates may follow such a one-step, instantaneous law, other processes such as mixing, vaporization, or a complex chemical reaction do not. When the characteristic times of these more complex processes are of the same order as the wave propagation time, important interactions occur.

After a transformation to a Lagrangian coordinate system, the author states that the linearity of the governing differential equation for the pressure outside of the combustion zone indicates that the pressure waves "propagate in a trivial way without growth, decay, or dispersion." This usually is not the case for ordinary gases. The linearity of the differential equations demands that the acoustic impedance (which equals the square root of minus the partial derivative of pressure with respect to specific volume at constant entropy) be constant with pressure or volume. This demand has been satisfied by the choice of the equation of state. This equation of state never is true for polytropic gases and is true only for such materials as solids which obey Hooke's Law. A realistic choice of a state equation would have shown a distortion of the waveform due to nonlinear effects and thus a possible shock formation. Once the shock has formed, one cannot say there is "no growth, decay, or dispersion." For example, the asymptotic behavior of the N wave is well known to be such that the shock strength decreases in inverse proportion to the square root of time, and the width of the wave increases as the square root of time.[2] Also, note that care must be taken in applying the transformation when a shock or detonation discontinuity is present, since the Jacobian of the transformation becomes discontinuous at such points. The transformation must be applied separately on each side of the discontinuity, and then the coordinates should be matched at the point of discontinuity.

Finally, and most important, the author has implied that, although boundary conditions may affect the stability criteria for certain practical configurations, the local interaction between a pressure wave and the combustion process is most important in determining the stability criteria. This generally is not the case, since dissipation phenomena introduced through the boundary conditions usually are as important as the forcing function introduced through the combustion process. For instance, in liquid rocket engine instability, there is a loss of oscillation energy by convection of the mean flow out the nozzle. Furthermore, oscillation impedance of the nozzle usually causes a reflected wave to have a lower energy than the incoming wave. Both of these effects are of the same importance as the forcing function, which is of the order of the mean flow. Therefore, in practical problems, one cannot look at local stability criteria but must concern himself with stability in the large.

References

[1] Rosen, G., "Stability of pressure waves in a combustion field," ARS J. **32,** 1605–1607 (1962).

[2] Courant, R. and Friedrichs, K. O., *Supersonic Flow and Shock Waves* (Interscience Publishers Inc., New York, 1948), pp. 164–168.

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Reply by Author to W. C. Strahle and W. A. Sirignano

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THE preceding comment by Strahle and Sirignano discusses two aspects of Ref. 1. Some practical limitations of the idealized mathematical model are mentioned. It then is asserted that local stability criteria should not be studied and that the stability problem is always essentially global in character, like a boundary-value or eigenvalue problem.

With regard to the practical limitations of the model, it should be pointed out that a more general form for the

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